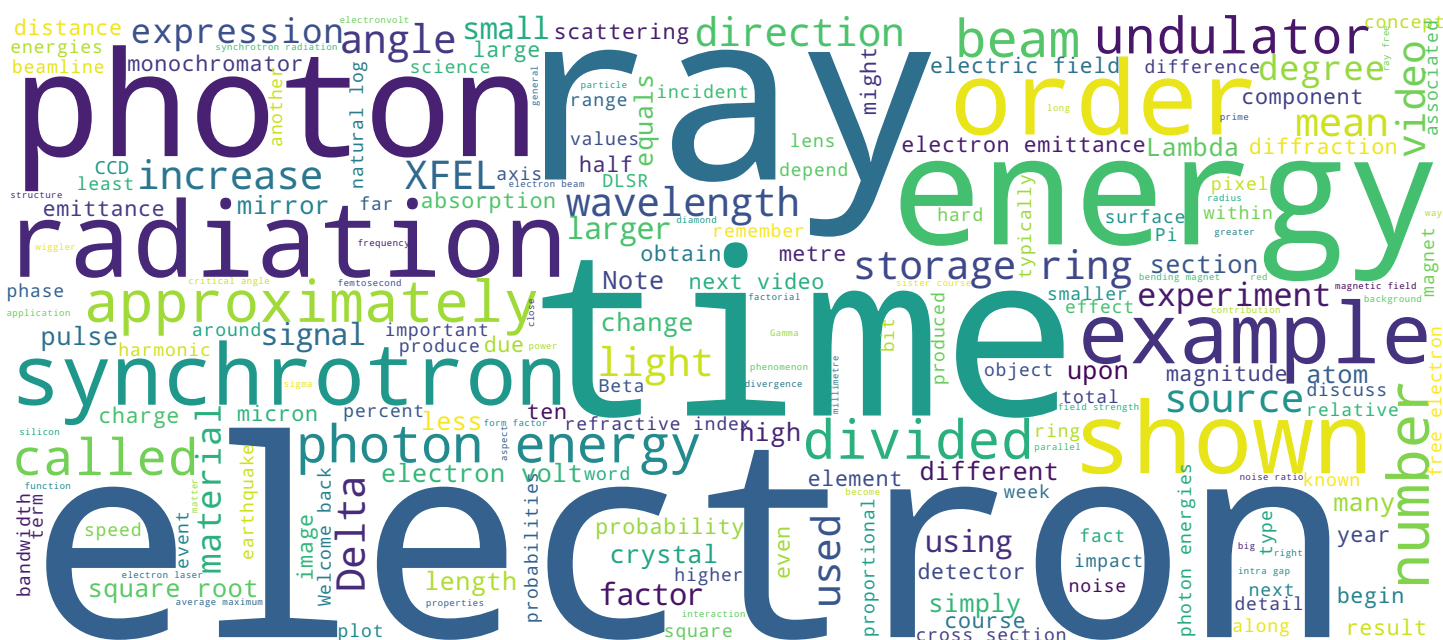


Prof. Philip Willmott



## Search MOOC



## Video



# Contents and objectives of this video



## ▪ Noise

- Poisson statistics and shot noise
  - Examples
- Dark noise
- Readout noise

Welcome back. We continue week six, the final week of this first of two sister courses, synchrotrons and X-ray free electron laser techniques and applications. By discussing in this last section photon and electron detectors. In this first video we look at the concept of noise and particularly its description, using Poisson and normal statistics. By normal, I refer to the well known Bell curve distribution described using Gaussian curves. We then look specifically at dark and read out noise.

Notes

Summary



0m 05s

# Poisson statistics



- Express probability of a given number of events occurring in a fixed interval of time or space
  - Requires a knowledge of the mean rate  $N$
  - Occurrence of one event has no influence on probability of other events occurring

Poisson statistics expresses the probability of a given number of events occurring in a certain interval of time or space, Assuming both that the mean rate after many such events is known, and that the occurrence of one event has no impact on the probability of any future event occurring. It is also assumed that no two events can occur exactly at the same time.

Notes

Summary



0m 44s

# Poisson statistics

- Probability of  $k$  events occurring in time  $t$ , whereby the mean number of events in time  $t$  is known to be  $N$ , is:

$$P(k) = \frac{e^{-N} N^k}{k!}$$

- For large  $N$ , this can be approximated by the “normal” distribution

$$P(k) \approx \frac{e^{-(k-N)^2/2N}}{(2\pi N)^{1/2}}$$

which has a standard deviation =  $N^{1/2}$

\* Stirling's (Willmott's) approximation:  $\ln n! \approx (n + 1/2) \ln n - n + \ln[(2\pi)^{1/2}] \approx (n + 1/2) \ln n - (n - 1)$

Precisely the probability  $P_K$  of  $K$  events occurring in a time  $T$  whereby the mean number of events in that time is known to be  $N$  is given by  $P_K = e^{-N} N^K / K!$ . This is a precise expression. Note that for large numbers for  $K$ , the value  $K!$  can swiftly become difficult to manage. I remember when I got my first scientific calculator back in the day it was about 1977. Sixty nine factorial was as far as you could go before the exponent exceeded the maximum of ten to the 99. Fortunately, there are several approximations to determine the natural logarithm of  $N$  factorial. The best of which seems to be natural log  $N$  factorial equals  $N$  plus a half, natural log  $N$  minus  $N$  plus natural log of two  $\pi$  to the half, which also approximates pretty well with,  $N$  plus a half natural log  $N$  minus  $N$  minus one. In any case, once  $N$  becomes a large number, the true platform distribution can be very accurately approximated by the normal Gaussian distribution.  $P_K$  is approximately equal to  $e^{-N} N^K / (2\pi N)^{1/2}$  or divided by  $2\pi N$  to the half. This has a standard deviation of plus or minus  $N$  to the half.

Notes

Summary



1m 15s

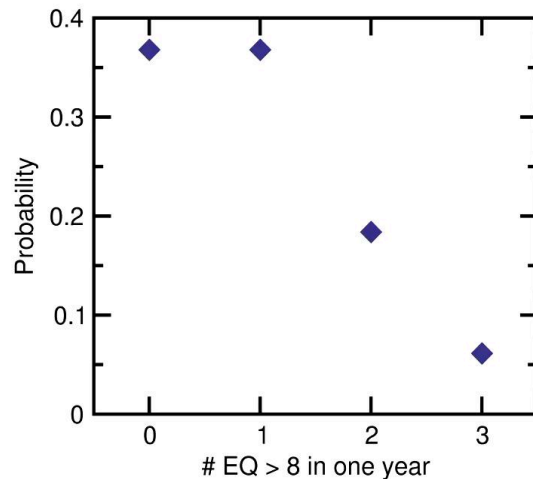
# Poisson statistics – examples

- Global average number of earthquakes with  $M \geq 8 = 1/\text{yr}$
- What are the probabilities for 0, 1, 2, and 3 such earthquakes in a given year?

- Solution:  $N = 1$   
 $k = 0, 1, 2, \text{ and } 3$

Use proper eqn. for  $P(k)$  as  $N$  is small

$$P(k) = \frac{e^{-N} N^k}{k!}$$



Okay, so let's look now at a couple of examples to both understand these expressions more clearly and to probe the relative accuracy of the more amenable normal distribution as a function of the average number of recorded events  $N$ . We begin with a simple example, the number of earthquakes per year with a magnitude of eight or greater. This has been found to be one per year after over a century of accurate data and another century before that of more approximate estimates. The data for the 19th century is still fairly reliable, as the damage wrought by a magnitude eight earthquake or larger is fairly easy to identify unfortunately. So with this information at hand, we can calculate using our expression for  $P(k)$  for the probabilities, for zero, one, two, and three such earthquakes occurring in a given year. It emerges that the probabilities for none and one earthquake are the same at a value of one upon  $E$ , which equals  $\text{knot}.368$ . The values for two and three are respectively  $\text{knot}.184$  and  $\text{knot point knot}61$ . The sum of these four possible outcomes is  $\text{knot}.981$ . All other possible outcomes thus have a probability altogether of less than 2 percent.

Notes

Summary



3m 07s

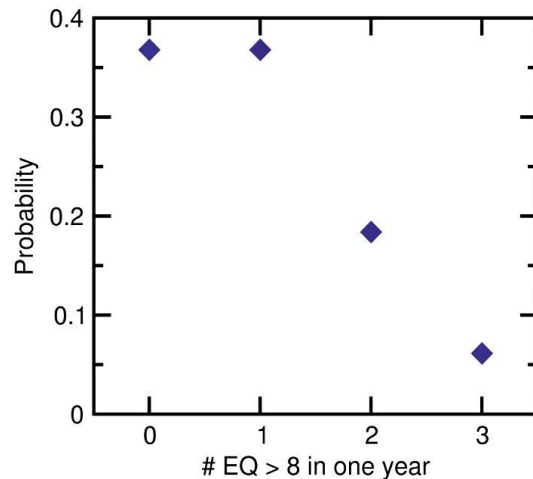
# Poisson statistics – examples

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$$P(k) = \frac{e^{-N} N^k}{k!}$$



One small point, this example of earthquakes was deliberately chosen as being perhaps suboptimal to underline the criterion that the occurrence of one event must have no impact on the timing of the next. We all know that one earthquake can trigger further seismic events after shocks and so on. Nonetheless, a chain of eight magnitude earthquakes is extremely rare.

Notes

Summary



4m 39s



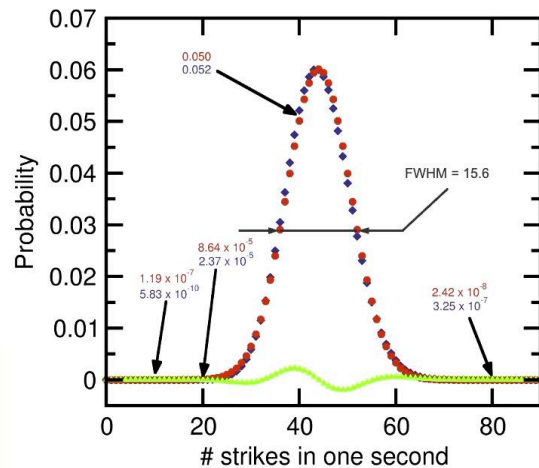
# Poisson statistics – examples

- Global average number of lightning strikes/s = 44
- What are the probabilities for 10, 20, 40, and 80 lightning strikes in a second?
- Solution:  $N = 44$   
 $k = 10, 20, 40, \text{ and } 80$

Use both proper eqn. for  $P(k)$  and normal distribution approximation

$$P(k) = \frac{e^{-N} N^k}{k!}$$

$$P(k) \approx \frac{e^{-(k-N)^2/2N}}{(2\pi N)^{1/2}}$$



Our second example concerns the number of lightning strikes globally, which amount to some 44 per second. We consider the probabilities on any given second, 10, 20, 40, and 80 strikes occurring. The probability for 10, 20 and 80 are very small indeed, lying as they do on the outer flanks of the real and approximate expressions. With approximately plus or minus one full width of half maximum, that is, plus or minus 16 however, the probabilities are significant and the two curves agree fairly well with one another. Note that the green curve shows the difference between the two expressions.

Notes

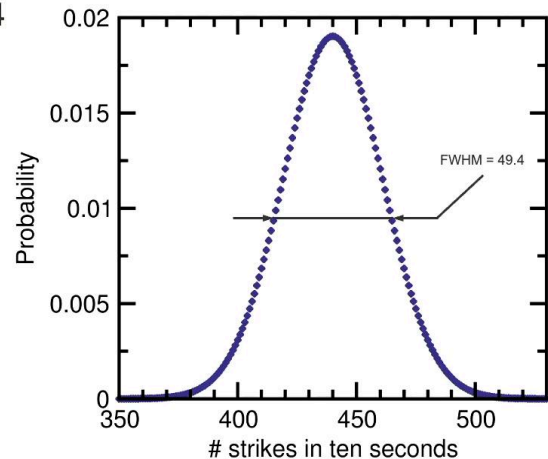
Summary



5m 06s

# Poisson statistics – examples

- Standard deviation of normal approximation  $\sigma = N^{1/2}$
- FWHM =  $(8 \ln 2)^{1/2} \sigma = 2.355 \sigma$ 
  - e.g. here: FWHM =  $2.355 \times 440^{1/2} = 49.4$
- $\sigma/N = 1/N^{1/2}$
- Take-home message:  
S/N ratio increases with square root of signal strength



We now consider the number of lightning strikes globally in a ten second interval, which we can expect to be on average equal to 10 times 44 or 440. The full width half maximum is not 156, 10 times that for the curve for the number of strikes for 1 second, but in fact only 49.4. This is in fact 15.6 multiplied by the square root of 10. Expressed differently the relative scatter of the result that is, the average divided by the standard deviation is now the square root of 10 times smaller than it was before.

Notes

Summary



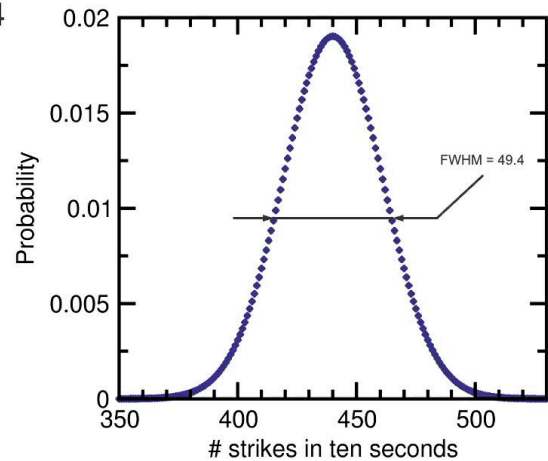
5m 53s



# Poisson statistics – examples

- Standard deviation of normal approximation  $\sigma = N^{1/2}$
- FWHM =  $(8 \ln 2)^{1/2} \sigma = 2.355 \sigma$ 
  - e.g. here: FWHM =  $2.355 \times 440^{1/2} = 49.4$
- $\sigma/N = 1/N^{1/2}$

Take-home message:  
S/N ratio increases with square root  
of signal strength



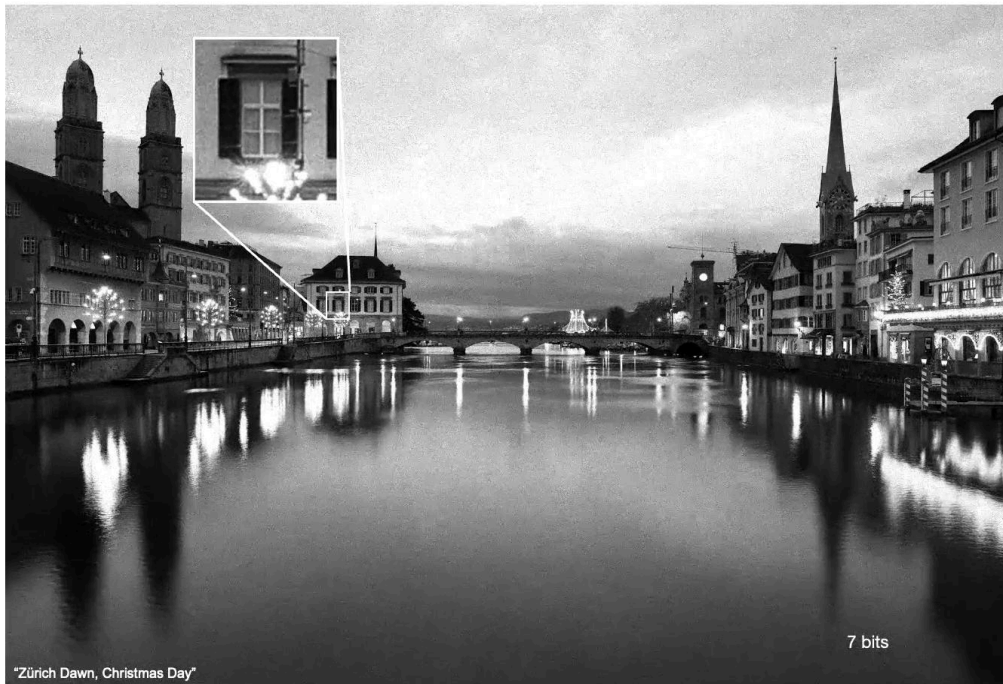
So the take home message is therefore that the accuracy of measurements or its signal to noise ratio improves with the square root of the measurement effort, eg, the length of time invested to make the same measurement.

Notes

Summary



# Poisson statistics



Original image courtesy of Nyah Willmott

So what is the consequence of this relationship between signal to noise ratio and measurement time? Shown here is a simulation of plus on noise in a photograph starting from an image with one bit intensity resolution and increasing all the way to 16 bits. So let's have a look at it. What does this mean? In the first frame on average, each Pixel either records no signal corresponding to black or a single event or photon equating to white. However, we add plus on noise, which means the probability of 0,1,2,3 and so on photons being recorded follows the same distribution as we showed earlier for the number of magnitude eight or greater earthquakes per year. Each subsequent frame increases the average maximum by a factor equal to the 10th root of two or 1.122. This means that after ten frames the average maximum is two bits, or simply two, while another 10 frames later it is three bits or four and so on and so forth. Each increase in the number of bits by one thus correspond to a doubling of the exposure time. Each image has been normalised so that the full range from black to white covers the range from zero to the maximum signal. By the time we have reached 16 bits or 32,768 average maximum intensity.

Notes

Summary

6m 53s



# Poisson statistics



"Zürich Dawn, Christmas Day"

Original image courtesy of Nyah Willmott

The relative maximum Poisson noise is one upon the square root of 32,768 or knot point knot knot552. Look now at the improvement in the quality as we look through the animation. For me, at least I see no real improvement beyond 13 bits, which means the last three bits corresponding to an eight fold increase in exposure time is really unnecessary. The blow up of the window in the top left is a good region to judge when the improvements become unobservable. Of course, what is considered unnecessary signals noise ratio depends very much on your goal. A professional photographer will want the 13 or better bit resolution. But if all you want to do is identifying the scenery, one or two bits are actually sufficient. We come back to this aspect in the next video.

Notes

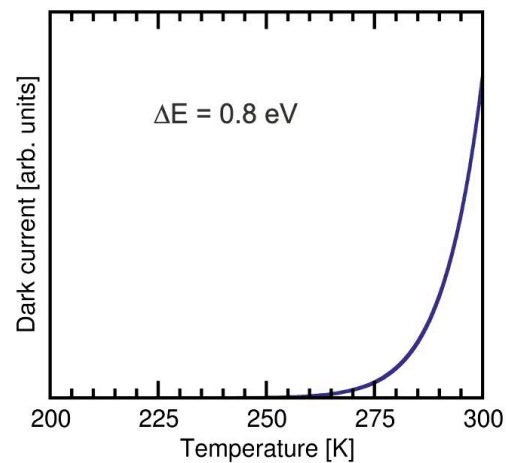
Summary



8m 33s

# Dark current and dark (or thermal) noise

- Ideally: no current in photosensitive devices if no photons
- Reality: dark current due to random generation of electrons and holes within depletion region of device
  - Caused by crystallographic defects in diode p-n junctions
    - Generate "intragap states" in semiconductor
  - Temperature dependent
    - Typically superlinear
- $$I_d = I_{d0} \exp(-\Delta E/kT)$$
  - Cool detector e.g. with Peltier device
- Important in charge-coupled devices (CCDs)
- Dark noise follows Poisson statistics
  - $\propto I_d^{1/2}$



Before that, however, we finished this video by discussing dark or thermal noise. In an ideal world, a photon detecting device should give no signal at all if kept in a completely dark environment, at least in the photon energy range it has been designed to detect. In many devices, however, there will be a background so called dark current due to the random thermal generation of electrons and holes within the depletion region of the device. Without going into the solid state physics behind this phenomenon, it is sufficient to say that this current is caused by crystallographic defects in diode PN junctions, which generate intra gap states in the semiconductor material. The current increases in a super linear fashion with respect to temperature that can be modelled by the expression,  $I_d$  equals  $I_{d0}$  multiplied by the exponent of minus delta E divided by K T, where delta E is the energy of the intra gap state above the valence band and T is the absolute temperature, k is of course, Boltzman's constant, 1.38 times 10 to the minus 23 Jules per Kelvin sorry. A plot is shown here. One sees that by cooling to a few 10s of degrees below freezing, the dark current can be effectively suppressed compared to its value at 300K or room temperature.

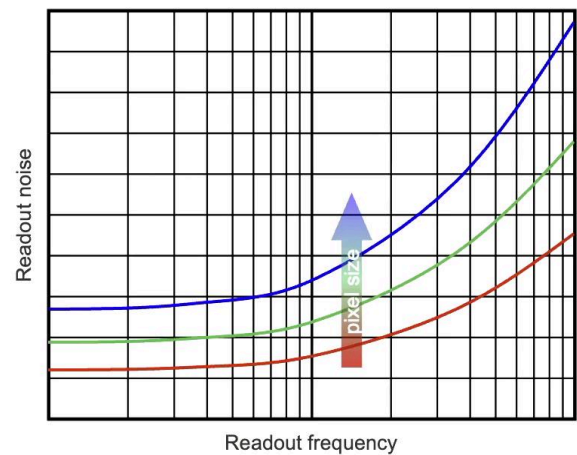
Notes

Summary



# Readout noise

- Three-step process in CCD detectors
  - Signal stored as electron charge  $q$  within each pixel
  - Convert to voltage using capacitor
    - $V = q/C$
  - Convert analogue voltage  $V$  to digital signal using A/D converter
- Readout noise happens in second step converting  $q$  to  $V$
- Relative noise increases with readout frequency



This effect is important in CCDs, which are often cooled to minimise dark noise. Dark noise follows Poisson's statistics for a given intra gap to energy and temperature. In CCDs, the signal generation follows a three step process. First, the light signal is converted to charge, which is stored within each Pixel. The charge is then converted to a voltage via a capacitor, then, in the third step converted to a digital output using an analogue to digital converter. In the second step, during the conversion from charge to voltage, another source of noise is generated called read out noise. This increases with both read out frequency and the Pixel size.

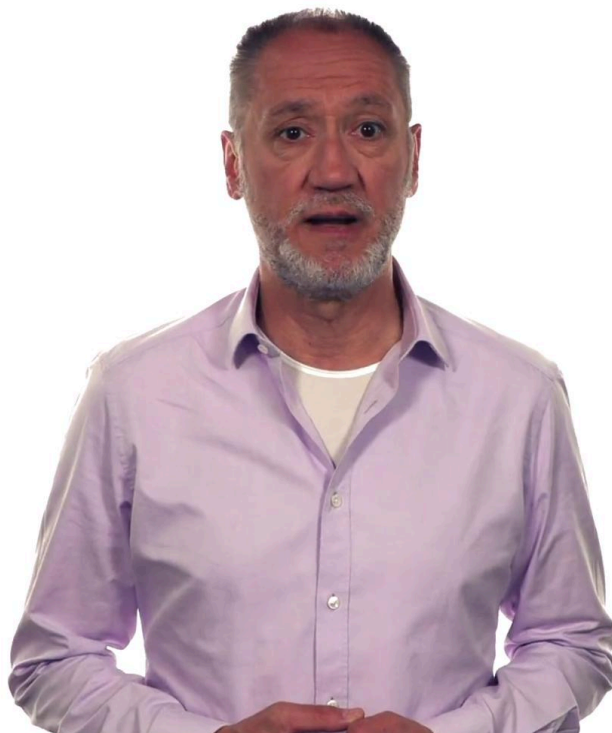
Notes

Summary



11m 13s

## In the next video...



In the next video, we look more closely at what we might expect an X-ray photon detector should do. And then we consider the problem of extracting a desired signal from a background of unwanted signal or noise. We also consider non-stochastic or systematic errors. And finally, we will take a brief look at flat field recordings and point spread functions.

Notes

Summary



12m 05s